Measurement of Two-Mode Discontinuities in a Multimode Waveguide by a Resonance Technique*

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Summary—The deliberate use of two or more propagating modes in a multimode waveguide, and a knowledge of associated control elements, has assumed renewed importance, particularly for millimeter wavelength applications. This paper presents a resonance measurement technique for the precise evaluation of the equivalent network for a lossless shunt discontinuity coupling two nondegenerate modes in a multimode waveguide. The discontinuity structure is placed into a cavity closed by adjustable plungers, and the data consists of those plunger positions which render the cavity resonant in the two modes of interest. This multipoint data is then transformed to permit an analysis of the two-port network in the discontinuity plane by conventional techniques.

Computations and experimental results obtained at *S* band illustrative of the procedure are presented for shunt discontinuities coupling the E_{01} and H_{01} modes in circular waveguide. The accuracy achieved is comparable to that obtained in single mode precision measurements.

I. Introduction

TECHNIQUES for the measurement of the equivalent network parameters of discontinuities coupling two or more modes in a multimode waveguide are of interest for a variety of guided wave systems admitting the propagation of more than one mode. Included among these are coupled closed or open (surface or leaky) waveguides supporting several "normal" modes, and oversized conventional waveguide configurations,[1–6] with renewed interest in the latter provided by millimeter wavelength applications. We may distinguish two broad categories of multimode waveguide operation: those which seek to restrict propagation to a single preferred mode, and those which exploit deliberately the presence of, and interaction between, several modes. The associated multimode measurement techniques r flect the needs of these waveguide systems. For singl mode operation, every effort is made to maintain moc purity, and the aim is to detect and minimize the pre ence of unwanted propagating modes. Thus, a measur ment of the (small) relative power coupled into tl spurious modes is usually sufficient.[2–5] For true mult mode operation, on the other hand, one is often co cerned with discontinuities which cause appreciable i terchange of energy between various modes, as utilize for example, in the design of mode transducers, dire tional couplers, filters,[6] etc. In this instance, a comple and accurate knowledge of the network properties of discontinuity is essential.

This paper describes a resonance technique for mea uring the complete equivalent circuit for a discont nuity. Although the method can be adapted, in princ ple, to measure lossless discontinuities coupling an art trary number of modes, its experimental utility is der onstrated herein only for shunt discontinuities coupli two modes of a multimode waveguide. In this procedu a cavity is formed by closing a section of uniform wav guide at both ends with adjustable short-circuiti plungers. The two mode coupling or discontinuity stru ture to be measured is inserted into this simple cavit the coupling structure may be thought of as a perturb tion of the original cavity. The data required for tl evaluation of the equivalent network parameters for tl discontinuity element are various plunger positio which render the perturbed cavity resonant in the tv coupled modes. The method employed for abstracti the circuit parameters from the basic data, reduces th data to that conventionally obtained in the single-moc measurement of a lossless two-port by the method sliding short circuits. Familiarity with the measureme of lossless two-ports by a movable short circuit tecl nique is assumed.[7–9]

The present scheme appears to offer a number of a vantages over other techniques which would rely on tl measurement of the amplitudes and phases of the r

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[8] N. Marcuvitz, "Waveguide Handbook," Radiation Lab. S McGraw-Hill Book Co., Inc., New York, N. Y., vol. 10, sec. 3 3.4, pp. 117–138; 1951.

[9] A. Weissfloch, "Schaltungstheorie und Messtechnik des De meter—und Zentimeter—Wellengebietes," Birkhauser Verlag, Ba und Stuttgart, pp. 195–200; 1954.

flected and transmitted waves in each mode.[1-3] First, since only a coupled mode resonance need be detected, it is not necessary to discriminate between the two modes concerned. This feature represents a considerable simplification in instrumentation and procedure. Second, the data required for the complete analysis of lossless discontinuities are merely the resonant plunger positions which can be measured simply and accurately. Third, multipoint data for shunt structures can be analyzed by averaging methods of the semiprecision or precision type familiar from the theory of single mode measurements,[7] thus leading to accurate results for the equivalent network parameters. This feature is particularly important if the discontinuity couples two modes out of a complex of other possible propagating modes, in which case, the measurement procedure must distinguish between resonances in the desired and the spurious modes. A precision analysis will highlight any inconsistencies in the experimental data.

The measurement technique proposed herein has been verified experimentally at the S band of frequencies in a circular waveguide cavity containing a shunt discontinuity coupling the E_{01} and H_{01} modes.

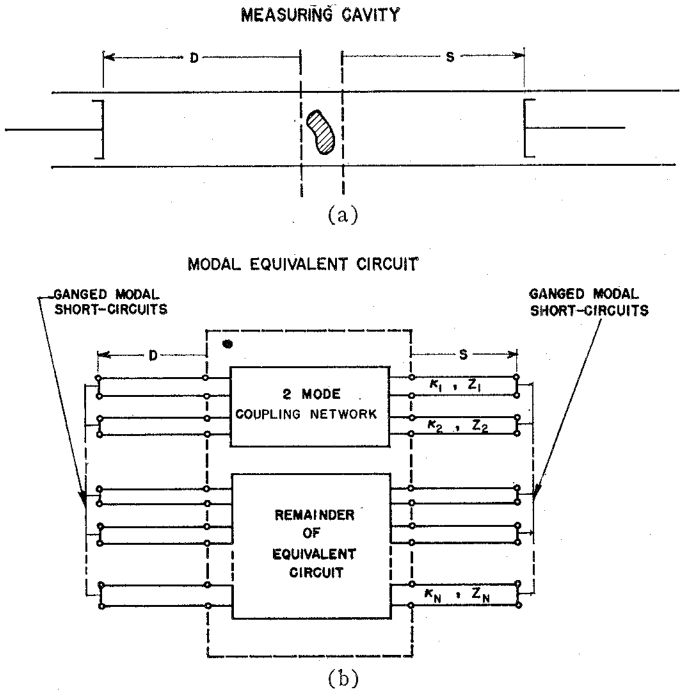

Fig. 1—Measuring cavity and equivalent circuit.

II. Network Description of Multimode Cavity

A. Remarks Concerning N-Mode Discontinuities.

1) The Resonance Relation: The structures considered in this paper comprise lossless discontinuities situated in a multimode uniform waveguide [Fig. 1(a)]. As is well known, the fields in regions far from the discontinuity structure may be expressed as a superposition of the propagating modes of the waveguide, and the modal coefficients may be interpreted as voltages and currents on equivalent uniform transmission lines, one for each mode, coupled only in the discontinuity region. The relationship between several or all of the propagating mode voltages and currents imposed by the presence of the discontinuity can be inferred from the network coupling the transmission lines, as shown in Fig. 1(b).

The measurement procedure for the evaluation of the parameters describing the coupling network is based on an analysis of the cavity formed when the multimode waveguide containing the discontinuity structure is closed by movable conducting plungers located "far" from the discontinuity [Fig. 1(a)]. The quantities D and S are physical lengths, measured from arbitrarily chosen reference planes to the plunger short circuits in the sense indicated by the arrow. In the equivalent circuit of Fig. 1(b), the plungers are represented by ganged modal short circuits, and the discontinuity region (included between the reference planes) by the dashed rectangle. The various modal transmission lines are characterized by propagation constants κ_i and characteristic impedances Z_i, $i=1, 2, \cdots, N$. The internal structure of the discontinuity network, shown by solid lines, is specialized for a two mode discontinuity as will be discussed further below.

The cavity formed by the plungers and waveguide is resonant if a finite field may be sustained in it without excitation. The condition at resonance is expressed by

$$\overleftarrow{B} + \overrightarrow{B} = 0, \tag{1}$$

where $\overleftarrow{B}$ and $\overrightarrow{B}$ designate the susceptance seen looking to the left and to the right, respectively, from a terminal pair chosen anywhere in the equivalent network.

2) The Resonance Diagram: When the positions of the cavity plungers D and S are plotted along cartesian axes, the contours which represent those particular plunger positions that render the cavity resonant constitute the "resonance diagram" of the cavity. In the absence of any discontinuity structure, the contours of the resonance diagram evidently consist of straight lines [see dashed lines in Fig. 2 for the case of only two propagating modes with guide wavelengths $\lambda_{g1,2}$, where the two reference planes in Fig. 1(a) have been chosen to coincide]. The introduction of a discontinuity into the cavity results in the perturbation of some of these lines. The simplest perturbation occurs when the discontinuity disturbs only one of the propagating modes so that only the lines corresponding to that mode are altered. The pertinent contours then assume the form characteristic of a two-port tangent relation curve familiar from the theory of measurement of a lossless two-port by a sliding short circuit technique.[8] The modification of the contours due to the introduction of a multimode discontinuity is much more complicated. Certain special features of these contours will be discussed in subsequent sections.

In the proposed measurement technique, the available data are those positions of the two plungers for

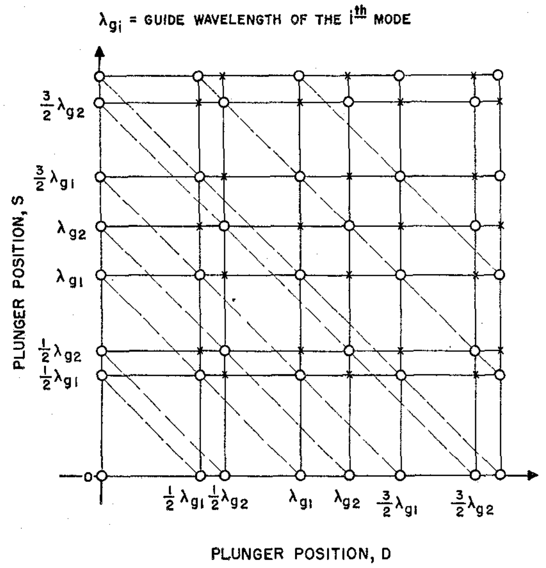

Fig. 2—Resonance diagram of empty measuring cavity.

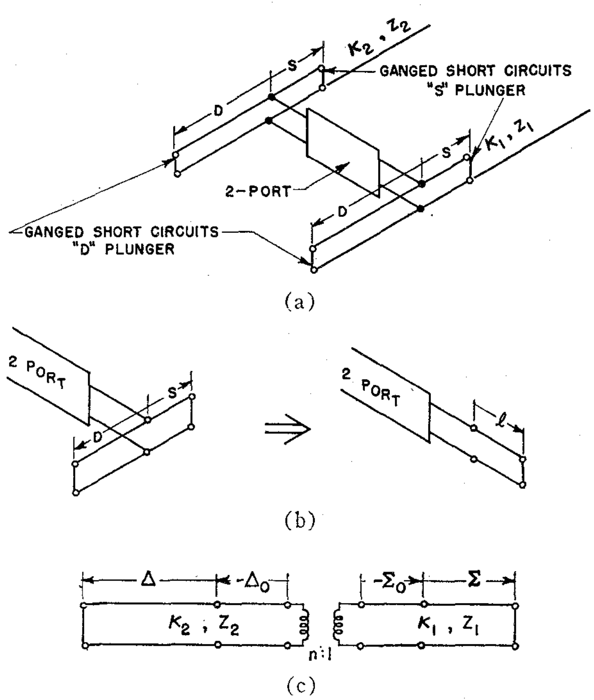

Fig. 3—Analysis of shunt two-mode discontinuities. (a) Equivalent circuit for pure shunt 2-mode discontinuity in measuring cavity. (b) Equivalent length $\cot Kl = \cot KS + \cot KD$. (c) Conventional 2-port analysis for discontinuity parameters.

which the cavity is resonant. Since the equivalent modal short circuits in Fig. 1(b) are not individually adjustable, the question arises as to whether this information suffices to determine the parameters for an arbitrary discontinuity structure. When the various propagating modes have incommensurable wavelengths, it can be shown that the resonant plunger positions provide as complete a set of data[10] as could be obtained if the positions of the modal short circuits on the individual transmission lines were controllable independently. When the guide wavelengths are commensurable, but not in the ratio of small whole numbers, one usually obtains sufficiently general data for the determination of the equivalent circuit.

3) Shunt Discontinuities: A thin transverse structure in a multimode waveguide is characterized by the continuity of the transverse electric field (*i.e.*, the mode voltages) across the discontinuity plane. The corresponding equivalent network is therefore of the type shown in Fig. 3(a), that is, pure shunt with respect to the various modal transmission lines. (Although the present remarks apply to N-mode structures, the figures are drawn for the special case, $N=2$, discussed in detail later.) For such transverse structures, it is convenient to measure the plunger positions relative to the discontinuity plane.

The resonance data for a $2N$-port, namely, the resonant plunger positions D and S of Fig. 3(a), may be reduced to equivalent short circuit data for the N-port network in the discontinuity plane by a procedure schematized in Fig. 3(b) (for $N=2$). The susceptances of the two sections of modal transmission line connected to the ith port of the coupling network may be added and the result interpreted as an equivalent single length l_i of short circuited line by the relationship

$$\cot \kappa_i l_i = \cot \kappa_i S + \cot \kappa_i D, \qquad i = 1, 2, \cdots, N, \quad (2)$$

where κ_i is the propagation constant for the ith mode.

B. Two-Mode Shunt Discontinuities.

1) The Resonance Relation: We now consider in greater detail the special case of a two-mode shunt discontinuity, which will concern us henceforth. A two-mode discontinuity is defined as one which couples two propagating modes to each other but to none of the remaining propagating modes which may exist in a multimode waveguide. Its equivalent circuit consists of two disjoint portions as illustrated in Fig. 1(b) (solid lines). For thin transverse discontinuities in a uniform waveguide, the two-mode coupling network may be represented as shown in Fig. 3(a), *i.e.*, a two-port connected in shunt at the discontinuity plane across the two modal transmission lines representing the coupled modes of interest.

Equivalent short circuit data for the two-port in the discontinuity plane may be computed from the observed resonant plunger positions as described in Section II-A, 3). The equivalent single line lengths, defined in (2), are designated in Fig. 3(c) by $l_1 \equiv \Sigma$ and $l_2 \equiv \Delta$ and determined by the relationships

$$\cot \kappa_1 \Sigma = \cot \kappa_1 S + \cot \kappa_1 D,$$
$$\cot \kappa_2 \Delta = \cot \kappa_2 S + \cot \kappa_2 D. \quad (3)$$

These corresponding lengths Δ and Σ constitute a set of data equivalent to that which would be obtained if the

[10] L. B. Felsen, Analysis of Circular Waveguide Modes, Second Quarterly Rep., Signal Corps Eng. Labs., R-503.6-56, PIB-433.6, Contract DA-36-039-sc-71235, sec. III A-2, pp. 2–15; November 27, 1956.

two-port network in the discontinuity plane were measured directly by a variable short circuit technique. This conventional two-port data is most directly interpretable if the network in the discontinuity plane is represented by the transformer type circuit in Fig. 3(c). Then the resonance condition (1) evaluated at the left transformer terminals, yields the tangent relation between Δ and Σ,

$$\tan \kappa_2(\Delta - \Delta_0) - \gamma \tan \kappa_1(\Sigma - \Sigma_0) = 0, \qquad (4)$$

where

$$-\gamma = n^2(Z_1/Z_2).$$

The parameters Δ_0, Σ_0 and γ of the equivalent circuit are readily obtained from a graph of (4) plotted from the measured sets of resonant Δ, Σ values.[8]

It may be noted that Σ and Δ can be eliminated between (3) and (4) to yield an analytical expression for the resonance curves on the two-mode resonance diagram. This is a rather involved transcendental relation between D and S; generally, the curves possess no simple periodicity in either D or S. A resonance diagram was computed for a simple network (see Section II-B, 4) in order to anticipate the type of curves to be obtained from an actual measurement; an experimentally determined diagram is presented in Fig. 11 in the last section.

2) *Fixed Points of the D, S Resonance Diagram:* The curves of a resonance diagram for any shunt discontinuity will always pass through certain "fixed points" of the D, S plane. These points correspond to resonant field distributions in which the transverse electric field at the discontinuity plane in either mode vanishes. The coordinates of the fixed points, indicated by the circles drawn on the grid of Fig. 2, are

$$D = m\frac{\lambda_{gi}}{2}, \qquad S = n\frac{\lambda_{gi}}{2}, \qquad i = 1, 2, \qquad (5)$$

where the λ_g's are the guide wavelengths of the relevant modes and m and n are integers. Considered as data, such points yield no new information.

3) *Resonance Curves for Special Structures:* Resonance curves for the special class of two-mode shunt discontinuities whose network representation in the discontinuity plane is such that a short circuit is seen at one port when the second port is short circuited, pass, in addition, through the points designated by crosses in Fig. 2, the coordinates of which are

$$D = m\frac{\lambda_{g1}}{2}, \qquad S = n\frac{\lambda_{g2}}{2}. \qquad (6)$$

This special class of networks is representable by a shunt susceptance plus a transformer in the discontinuity plane. The field distribution associated with this resonance is somewhat peculiar: finite fields exist only in one mode to the left of the discontinuity plane and only in the other mode to the right of that plane.

One particular group of resonance curves obtainable under special conditions deserves some attention mainly to point out that these curves, or others nearly equivalent to them, should be avoided in experiment since little information concerning the network in the discontinuity plane can be abstracted from them. The conditions referred to are: a) that the two modes have commensurable wavelengths, and b) that the network in the discontinuity plane be represented by a two-port for which an open circuit is seen at one port when the second port is open-circuited. This implies that the network comprises a series reactance plus a transformer. In the absence of any discontinuity, condition a) permits separate resonances in the two modes to exist simultaneously when

$$D + S = m\frac{\lambda_{g1}}{2} = n\frac{\lambda_{g2}}{2}, \qquad (7)$$

where

$$\frac{\lambda_{g2}}{\lambda_{g1}} = \frac{m}{n}, \qquad m, n = \text{integers}.$$

A discontinuity introduced into the cavity and described by condition b) will not disturb these resonant plunger positions and the corresponding curves on the resonance diagram for the discontinuity will appear in unperturbed form (straight lines). The special cases mentioned are illustrative, and the discussion is not intended to be exhaustive.

4) *Sample Calculations of Resonance Data:* To illustrate the nature of the resonance curves to be obtained from the proposed cavity measurement, a set of D, S resonance data was computed for the following assumed simple network: a transformer directly coupling the two transmission lines [Fig. 4(a)]. The resulting curves, together with relevant numerical values, are shown in Fig. 4(b) and serve to exemplify some of the remarks made earlier in this section. The curves are nonperiodic and pass through the fixed points defined by (5) and (6). The modal guide wavelengths chosen for the computations have a ratio of nearly 3:2, and since the transformer coupling network is included in the class discussed above some of the resonance curves are approximately straight lines. In particular, the fourth curve of Fig. 4(b) satisfies approximately the condition that $D+S=\lambda_{g1}=(3/2)\lambda_{g2}$.

III. Measurement Procedure in Circular Waveguide

A. E_{01}-H_{01} *Mode Discontinuities.*

The measurement technique described above has been applied to the measurement of two-mode discontinuities coupling the E_{01} and H_{01} modes in circular waveguide. For reference, the field configurations of the E_{01} and H_{01} modes are reproduced in Fig. 5. Most important, both modes possess complete rotational symmetry about the guide axis. The cutoff wavelengths λ_c of the first ten (first seventeen, counting orthogonal

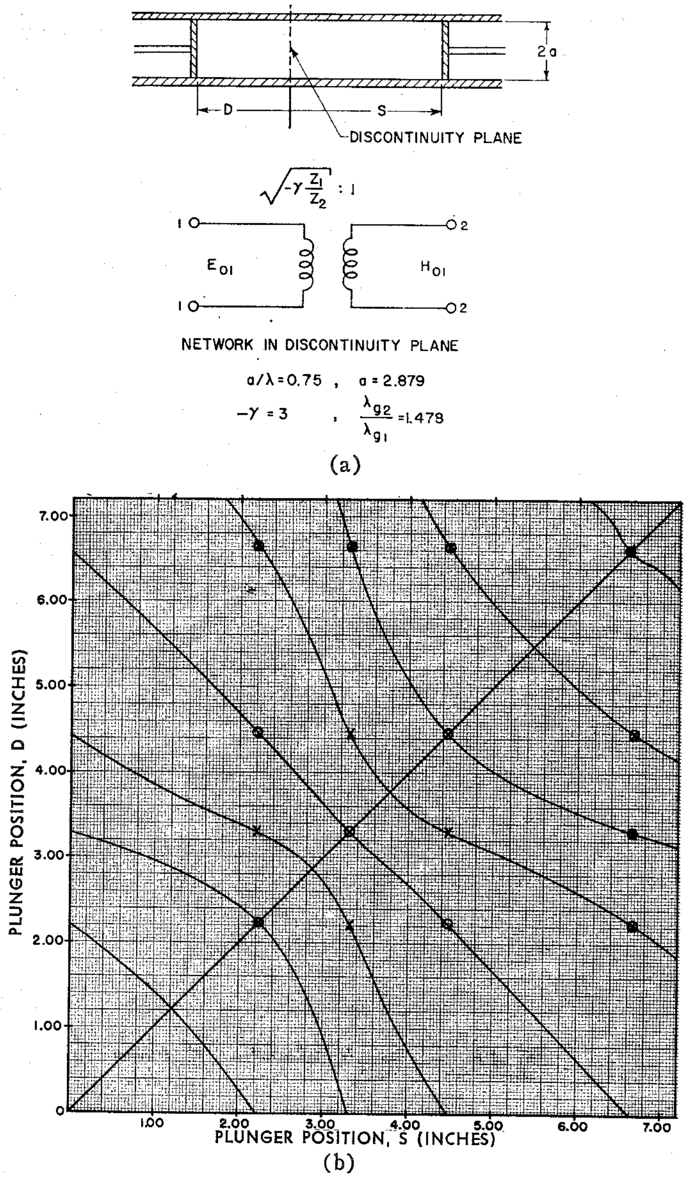

Fig. 4—Sample calculation of resonance data.

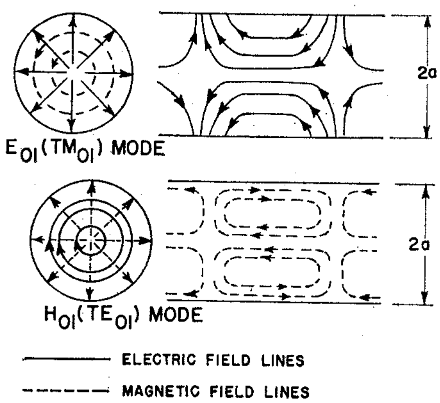

Fig. 5—Field patterns of the E_{01} and H_{01} modes of circular waveguide.

TABLE I

Mode	λ_c/a*
H_{11}	3.412
E_{01}	2.613
H_{21}	2.057
H_{01}, E_{11}	1.640
H_{31}	1.496
E_{21}	1.223
H_{41}	1.182
H_{12}	1.178
E_{02}	1.138

* λ_c is the cutoff wavelength and a is the guide radius.

polarizations) modes of circular waveguide are given in Table I.

Discontinuities which may couple the E_{01} and H_{01} modes to each other but to no other propagating mode in the waveguide may be obtained from the following considerations of symmetry. If the guide axis is an R-fold rotation axis for the discontinuity then, in view of the symmetry of an incident H_{01} or E_{01} mode, the scattered modes (E_{mn}, H_{mn}) must have azimuthal mode indexes such that

$$m = kR, \qquad k = 0, 1, 2, \cdots. \tag{8}$$

It is desired that the discontinuity scatter into only two propagating modes, *i.e.*, the E_{01} and H_{01} modes. On inspection of Table I, it is seen that it is sufficient to take $R \geq 5$ and to restrict the guide radius-to-wavelength ratio such that the E_{02} mode is cut off. In order to couple the E_{01} and H_{01} modes to each other, the discontinuity structures must not possess reflection symmetry with respect to any plane containing the waveguide axis. Typical shunt, *i.e.*, thin transverse, E_{01}-H_{01} discontinuities are illustrated in the upper right hand corner of the photograph, Fig. 6.

B. Description of Measuring Equipment.

Measurements of two-mode discontinuities coupling the E_{01} and H_{01} modes have been carried out at the S band of frequencies in 5.758″ ID (Nominal 6″) circular waveguide. A photograph and block diagram of the resonance measuring system are shown in Figs. 6 and 7. The system comprises three main assemblies: the microwave source assembly, the frequency monitoring assembly, and the measuring cavity assembly. Only the cavity assembly will be described in any detail since the other units are composed of standard commercial components.

The measuring cavity assembly consists of the cavity to be brought to resonance, a number of probes for sampling the electromagnetic fields in the cavity, and an amplifier system for the sampled signal. The measuring cavity, constructed in two half-sections, is based on a similar design by Sheingold.[11] One half-section, shown

[11] L. S. Sheingold, An Experimental Investigation of the Transmission Properties of the Dominant Circular—Electric Mode, Cruft Lab., Harvard Univ., Cambridge, Mass., Tech. Rep. No. 167; September 1, 1953.

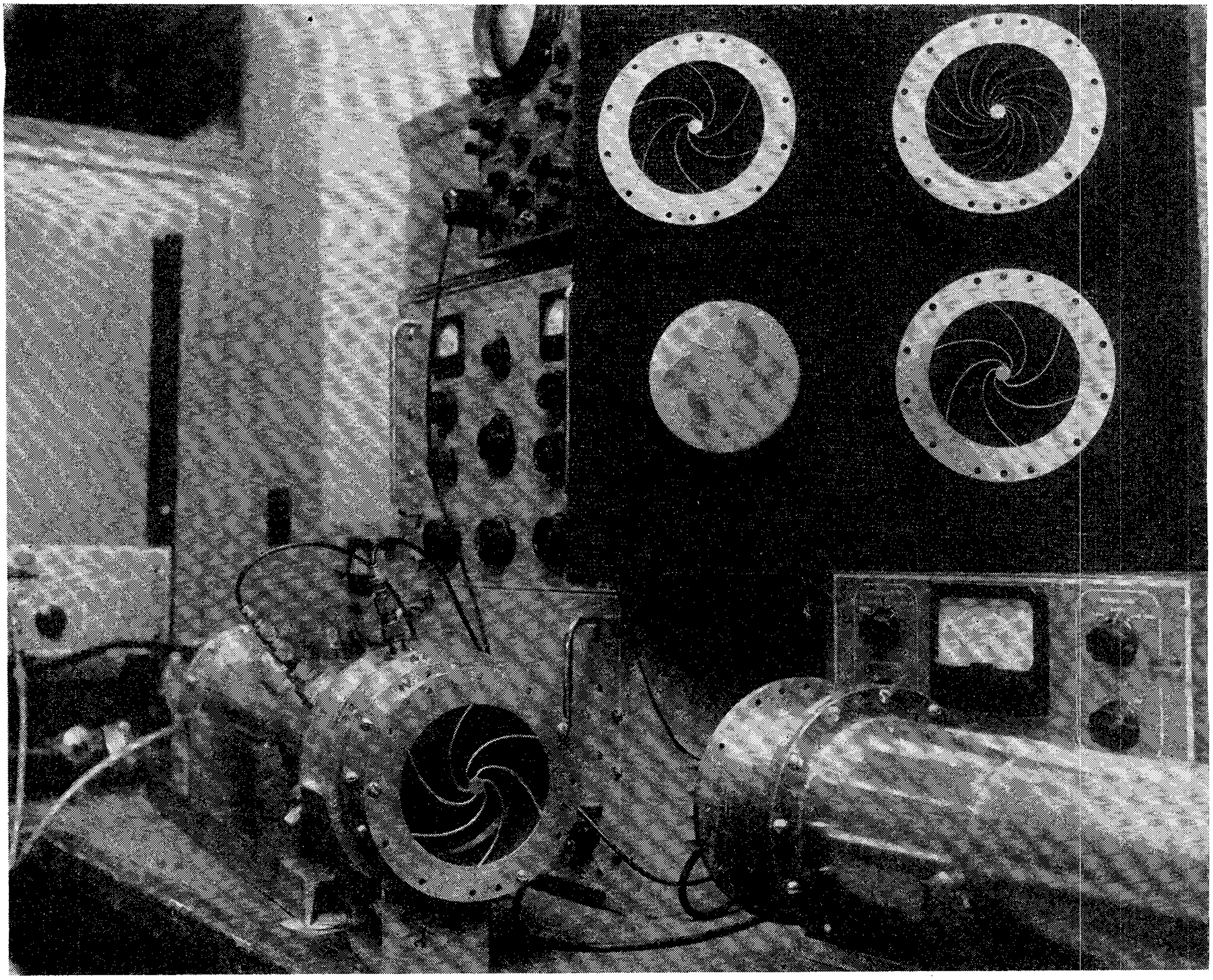

Fig. 6—Measuring cavity and typical E_{01}-H_{01} shunt discontinuities.

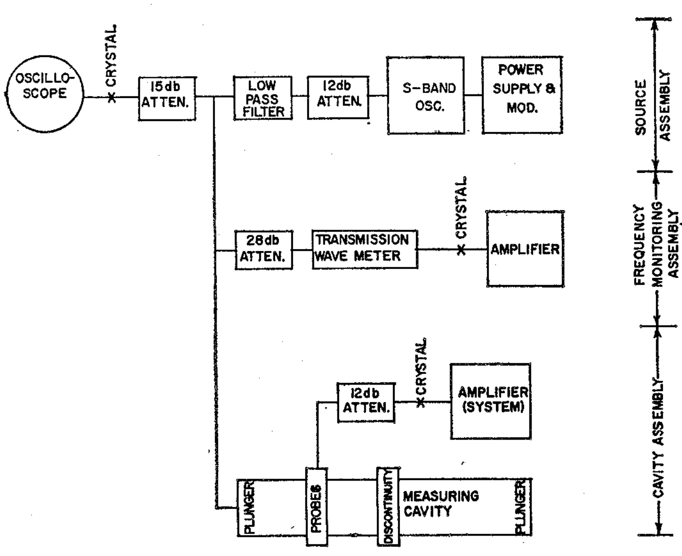

Fig. 7—Block diagram of the resonance measuring system.

in Fig. 8, consists of a length of brass pipe closed at one end by a contacting plunger, provided with a flange at the other end, and equipped with measuring and exciting probes; the other half-section differs only in that it has no provision for probes. Discontinuities to be measured are inserted at the flange between the two halves of the cavity. The short circuiting plungers make contact with the guide walls by means of compressed wire gaskets woven on Monel alloy. Good contact is required to join the radial currents induced by the E_{01} mode on the plunger face with the longitudinal currents induced on the guide walls. A hemispherical brass boss is centered on each plunger face to accentuate the difference between the susceptances presented by the plunger to the E_{11} and H_{01} modes, which have the same cutoff wavelength (Table I). The difference in plunger susceptance is the only feature causing the empty cavity to become resonant in these two modes at distinct

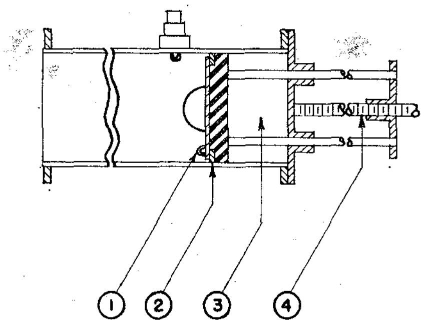

Fig. 8—Cross-section of the measuring cavity.

plunger positions. In the range of interest (frequencies lying between the H_{01} mode and the E_{02} mode cutoff frequencies), the symmetrical boss does not couple the E_{01} and H_{01} modes to each other or to any other propagating mode. The plungers are positioned by a precision drive screw with pitch 0.050″/revolution; an automatic revolution counter and vernier are attached.

The probes employed are shielded magnetic loops; a mechanically interchangeable electric probe was also constructed. The probe mounted in one plunger face is connected to the source, and the plane of all loop probes can be rotated in position. A separate sub-assembly carrying a probe which, in addition, can be moved azimuthally through a 90 degree sector is also incorporated.

C. Identification of Modes at Resonance.

The measuring cavity may be brought to a resonance involving one or more of the propagating modes by an adjustment of either the frequency or the plungers. Resonance is indicated experimentally by maximum transmission from the source to a detector probe through the cavity. In the process of obtaining the E_{01}-H_{01} resonance diagram, the pertinent resonances are easily identified by rotational symmetry of the combined modal fields. However, in the location of modal reference planes as described in the next section, it is necessary to distinguish between the E_{01} and H_{01} modes, themselves. The azimuthally movable probe provides the most direct way of identifying the circularly symmetrical modes. The probe is responsive to the tangential magnetic field at the guide wall, and the absence or presence of any variation with azimuthal position is indicative of resonance in a symmetrical or nonsymmetrical mode, respectively. In order to distinguish E_{01} and H_{01} resonances in the empty cavity, one may rotate the plane of the magnetic loop. A response for transverse orientation of the plane of the loop distinguishes an H mode.

Alternatively, the nondegenerate uncoupled modes in the empty cavity may be distinguished by the change in the position of a cavity plunger necessary to restore the cavity to resonance for a small change $\delta\lambda$ in source wavelength. The required fractional change $\delta(L/\lambda_g)$ in the resonant plunger position is given, in view of the relations.

$$\frac{d\lambda_g}{d\lambda} = \left(\frac{\lambda_g}{\lambda}\right)^3 \quad \text{and} \quad \frac{2L}{\lambda_g} = n, \quad n = 1, 2, \cdots, \tag{9}$$

by

$$\frac{\delta\left(\dfrac{L}{\lambda_g}\right)}{\delta\lambda} = \frac{n}{2\lambda_g}\left(\frac{\lambda_g}{\lambda}\right), \tag{10}$$

where L is the length of the cavity at resonance. Comparison of a measured ratio $\delta(L/\lambda_g)/\delta\lambda$ with the right hand side of (10) computed for the various possible modes leads to the identification of the resonant mode.

D. Calibration of Cavity for Effective Modal Short Circuit Positions.

The data required for the analysis of the discontinuity are the electrical distances at resonance from the discontinuity plane to the generally not coincident modal short circuits in the E_{01} and H_{01} modes. The usual experimental procedure for the location of the appropriate reference plane combines an approximate mechanical with a precise electrical measurement. A flat short circuiting plate is attached to the flange of that section of the empty cavity containing the source probe. At the desired frequency, the plunger is moved until the cavity section is brought to resonance in the mode for which the effective short circuit position is to be determined, and the dial indication [designated by $\tilde{D}_R$ in (11) below] corresponding to the cavity plunger position is noted. The precise distance from the short circuiting plate to the effective modal short circuit is then $n\lambda_g/2$, where λ_g is the corresponding guide wavelength. The integer n is determined from an approximate mechanical measurement of the cavity section length, or from (10). For the location of the analogous reference plane $\tilde{S}_R$ in the second half of the cavity, one connects the second half to the first half, sets the plunger in the latter at position $\tilde{D}_R$ and varies the plunger in the former until resonance is achieved. The reference plane locations $\tilde{D}_R$ and $\tilde{S}_R$ are required at every operating frequency and cannot be predicted from a single frequency measurement since the variation of the equivalent reactance of the plunger is not known.

If $\tilde{D}$ and $\tilde{S}$ denote the indicator readings for the resonant plunger positions taken during a measurement, the

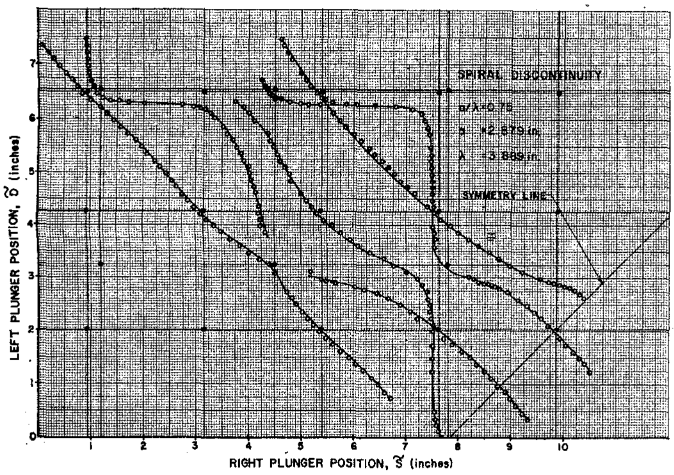

Fig. 9—Experimental resonance diagram, spiral discontinuity.

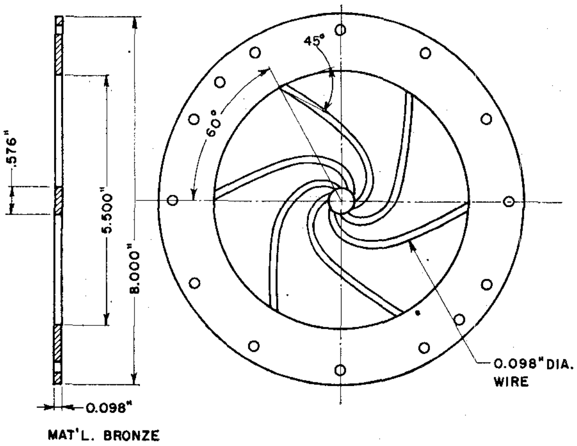

Fig. 10—Spiral E_{01}-H_{01} mode discontinuity.

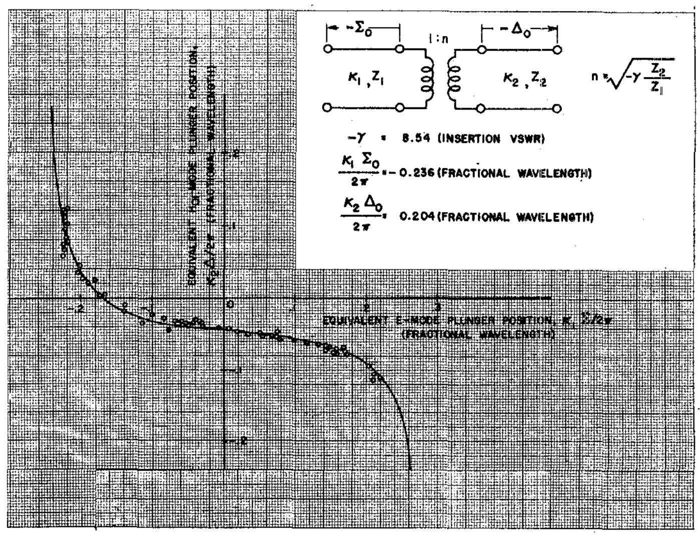

Fig. 11—Tangent curve for two-port in the discontinuity plane, (spiral discontinuity).

desired electrical distances $\kappa_{1,2}D$ and $\kappa_{1,2}S$ in (3) are then replaced as follows:

$$\kappa_{1,2}D = \kappa_{1,2}(\tilde{D} - \tilde{D}_{R_{1,2}}), \qquad \kappa_{1,2}S = \kappa_{1,2}(\tilde{S} - \tilde{S}_{R_{1,2}}), \quad (11)$$

where the subscripts $_{1,2}$ distinguish the two modes in question.

E. Special Techniques and Experimental Results.

Methods for recognizing when the cavity is resonant in the coupled (E_{01}, H_{01}) modes, and the calibration procedure linking observed plunger positions $\tilde{D}$, $\tilde{S}$ with the D, S values required in the computation of the discontinuity have been presented; it remains to describe any special techniques and experimental difficulties related to obtaining a set of $\tilde{D}$, $\tilde{S}$ values and to exhibit typical experimental results.

Although only three resonance measurements are needed, in principle, for the determination of the three parameters Σ_0, Δ_0, γ specifying the E_{01}-H_{01} mode coupling network, a precise evaluation employing an averaging scheme requires a larger number of data points. In practice, it is tedious to identify a given resonance as a desired one in the E_{01}-H_{01} modes, as distinct from a spurious resonance arising from other propagating modes. A simpler procedure is to start from a known E_{01}-H_{01} resonance, as, for example, from one occurring at a fixed point $\tilde{D}_R \pm n\lambda_g/2$, $\tilde{S}_R \pm n\lambda_g/2$ [see (5)], and to track that resonance as $\tilde{D}$ changes slightly, with a corresponding compensation in $\tilde{S}$. It was found extremely helpful to record data points $\tilde{D}$, $\tilde{S}$ graphically as illustrated in Fig. 9 in the progress of measurement.

When spurious mode resonances occur at nearly the same plunger positions as desired ones, the appreciable spurious mode amplitudes may influence the measurement in two ways: a) they may obscure desired resonances, and b) they may tune any small coupling due to asymmetries or other imperfections existing between network sections of Fig. 1(b), and thus appreciably affect the desired mode fields. Overlap of desired and spurious mode resonances is minimized by the high (loaded) Q of the measuring cavity. Plotting the observed data in the progress of the measurement aids in circumventing a).

The experimental resonance diagram in Fig. 9 was obtained for the spiral E_{01}-H_{01} mode discontinuity. This spiral discontinuity structure, shown in Fig. 10, consists of six symmetrically disposed logarithmic spirals which are defined by the requirement that, at every point, the spiral makes an angle of 45 degrees with a guide radius drawn through the point. This type of spiral was suggested by the fact that a small elliptical obstacle couples the E_{01} and H_{01} modes most strongly if placed with its major axis at 45 degrees to the unperturbed E_{01} and H_{01} mode transverse electric fields. The measurements were carried out at 3.075 KMc corresponding to a guide radius-to-wavelength ratio, $a/\lambda = 0.75$. This resonance diagram illustrates the nature of the resonance curves and actually contains considerab'y more extensive data than that required for the determination of the discontinuity parameters. It may be observed that the resonance curves pass close to, rather than through, the "fixed points" computed from calibration measurements. This may be attributed to the finite thickness of the discontinuity.

From each point $\tilde{D}$, $\tilde{S}$ on the resonance diagram, a point Σ, Δ on the tangent curve for the network in the discontinuity plane may be computed via (3) and (11). Fig. 11 shows this result for points chosen from all the curves of Fig. 9; only data from the immediate neighborhood of the "fixed points" have been excluded because of the thickness effect noted previously. The final test of the internal consistency of the two-mode dat was a precision analysis of the tangent curve,[7] whic will be described briefly. Values Δ' corresponding measured values of Σ were computed from the tange relation employing the parameters Σ_0, Δ_0, γ listed c Fig. 11. The differences $(\Delta'-\Delta)$ between the measur and computed values are a measure of the accura with which the parameters Σ_0, Δ_0, γ represent the e perimental data or the internal consistency. For t data presented, the measure of accuracy, $|\Delta'-\Delta|/\lambda$ <0.005, is comparable to values attained in single mo precision measurements.

Mode Couplers and Multimode Measurement Techniques*

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Summary—**The measurement of harmonic and spurious signals in waveguide systems is complicated by the fact that one must usually deal with a multimodal measurement. Since the energy may propagate in any mode consistent with the frequency and waveguide geometry, the measurement system used must discriminate between these different modes.**

A simple and direct approach to this problem is through the use of "mode couplers" which couple selectively to any desired mode. Theoretical and practical details for mode couplers for the first five modes in rectangular waveguide are presented, as well as the application and limitations of this measurement technique.

Introduction

The measurement of spurious and harmonic signals in waveguide systems is usually complicated by the fact that one must work with a multimode system. The energy one wishes to measure can be propagated in every mode consistent with the frequency and waveguide geometry. For example, a 6000-mc signal in a standard S-band waveguide ($3\times1\frac{1}{2}$ inches) can travel in the TE_{10}, TE_{20}, TE_{01}, TE_{11}, and TM_{11} modes. The distribution of power among these modes will, of course, depend on how the guide is excited.

To specify more exactly the nature of the problem, assume that the total power carried in an n-mode system is to be measured. To solve this problem n couplers are required, the output of each coupler being a linear function of the amplitude of each mode. Thus:

$$E = k_1M_1 + k_2M_2 + \cdots k_nM_n. \qquad (1)$$

A series of n amplitude and phase measurements usi these couplers will therefore yield the following set equations:

$$\begin{aligned}
E_1 &= k_{11}M_1 + k_{12}M_2 + \cdots k_{1j}M_j + \cdots k_{1n}M_n \\
E_2 &= k_{21}M_1 + k_{22}M_2 + \cdots \\
&\vdots \\
E_r &= k_{r1}M_1 \quad \cdots \quad k_{rj}M_j \\
&\vdots \\
E_n &= k_{n1}M_1 \quad \cdots \quad k_{nn}M_n. \quad (
\end{aligned}$$

Theoretically it should be possible to solve for t strength of the different modes from these equatio but clearly, where more than two or three modes a involved, the solution of such a set of complex equatio will be a tedious task. An alternative is to reduce t value of the cross-coupling terms k_{rj} to a value so lo that the output of each coupler is essentially depende on only one mode. This approach also greatly simplifi the measurement technique since the need for pha measurements is eliminated.

Mode Couplers

The basic mode coupler is shown in Fig. 1. It consis of two parallel waveguides mutually coupled throu two small apertures. The signal to be measured traveling from left to right in the lower or prima waveguide. The upper or secondary waveguide co

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